International Trade and Macro: Solving PE sunk-cost models

Solving discrete models

- Goal: Solve and estimate a sunk-cost model
  1. Solve model
  2. Estimate model
- Focus on steps 1–3 for now

Algorithm

1. Initial set up
   - Set parameter values
   - Construct grids; Discretize continuous stochastic processes
   - Initialize policy and value functions
2. Solve decision problem
   - Value/policy function iteration to convergence
   - Key output: Policy functions
3. Create simulated panel of data
   - Set initial firm states; Draw sequences of shocks
   - Use policy functions to model firm behavior, record panel
   - Use panel to compute moments in simulated data
4. Compare model-moments to data-moments
   - If moments match, finished
   - If moments do not match, update parameters, return to step 2.

1. Initial setup

- Parameters
  - θ = elasticity of substitution in demand
  - τ = tariff (constant for now, could be stochastic)
  - β = discount factor
  - δ = survival probability
  - f_0, f_1 = export entry, continuation costs
  - A process for z ($\bar{z}$, ρ, σ_ϵ)

$$\log(z') = (1-\rho)\log(\bar{z}) + \rho\log(z) + \epsilon$$

$$\epsilon \sim \text{iid } N(0, \sigma_\epsilon)$$

  - $\xi_H > \xi_L$ export variable costs (constant for now, could be stochastic)

1. Initial setup

- Construct a grid for z
  - Equally spaced points
  - Importance-weighted: Use CDF of ergotic distribution
- Use a Tauchen-like method to convert AR(1) to discrete Markov chain
- Initialize value and policy functions
  - $V^1(x, z, \xi)$ value function for exporter ($N_z \times N_\xi \times 2$)
  - $V^0(x, z, \xi)$ value function for non-exporter
  - $V(x, z, \xi)$ value of the firm (need two of these, old and new)
  - $X(x, z, \xi)$ export decision
- Initialize V to something like $(1 - \beta)\pi(z, \xi)$
- Precompute and store $\pi(x, z, \xi)$
- Ancillary functions: $l(x, z, \xi), ex(z, \xi)$. Compute after iteration converges.

2. Solve decision problem

- Value function iteration. Loop over z_i

$$V^1(x, z_i, \xi) = \pi(1, z, \xi) - xf_1 - (1-x)f_0 + \beta \sum_{z_j} V_{\text{old}}(1, z_j, \xi)\text{prob}(z_j|z)$$

$$V^0(x, z_i, \xi) = \pi(0, z, \xi) + \beta \sum_{z_j} V_{\text{old}}(0, z_j, \xi)\text{prob}(z_j|z)$$

$$V_{\text{new}}(x, z_i, \xi) = \max\left\{V^1(x, z_i, \xi), V^0(x, z_i, \xi)\right\}$$

- Check: $\| V_{\text{new}}(x, z_i, \xi) - V_{\text{old}}(x, z_i, \xi)\|$
- If not converged, set $V_{\text{old}}(x, z_i, \xi) = V_{\text{new}}(x, z_i, \xi)$, repeat
- Once converged, compute $X(x, z_i, \xi)$, $I(x, z_i, \xi)$, $ex(z_i, \xi)$

3. Simulate a panel

- We have the decision rules...
- Want to create a panel data set of firms in the stationary distribution
  1. $t = 0$: Create N_f firms, assign each a ξ and a z_0; all nonexporters
  2. $t = 1, \ldots, t = T$; $f = 0, \ldots, N_f$
     - Draw a z_t for firm f (use ergodic dist and uniform random)
     - Compute export decision, production, exports, etc.
  3. To avoid initial conditions problem, throw out first several hundred observations. Check that moments do not change (much) over the panel.
- Now we have a panel of data...
- If we structured out panel correctly we can **literally** use the same code we used on the data on the model panel.

Aggregate shocks

- No aggregate uncertainty here
- Make E_t an AR(1) process that affects all firms identically
- Need to discretize and add to the firm's state variables

$$V(x, z, \xi, E) = \pi(z, \xi, E) + \cdots + \sum_{z', E'}$$

- Easy to do in partial equilibrium; will typically overstate the effect of a foreign demand shock — price dynamics will attenuate