Two-country business cycle models w/production

UW Madison // Fall 2022

Roadmap

- Will mostly follow Backus, Kehoe, and Kydland (1994)

1. Facts
2. Model
3. Calibration
4. Success and failures

Data

- Business cycle analysis focuses on volatility and comovement
- Variables are real unless stated otherwise
- Log and HP filter data (still a good idea?)
- Typical measures
  - Volatility: standard deviation (percent)
  - Persistence: autocorrelation
  - Comovement: correlation

Stylized facts: closed economy

1. Output is less volatile than consumption (consumer durables?)

- $\sigma(c) < \sigma(y)$

2. Investment is much more volatile than output

- $\sigma(y) \times 3 \approx \sigma(x)$

3. Output is more volatile than productivity (need amplification)

- $\sigma(z) < \sigma(y)$ [measure z as Solow residual]

4. Hours worked varies across countries

- Typically, $\sigma(\ell) < \sigma(y)$

5. z, ℓ, c, x are procyclical

- As measured by correlation: e.g. $\rho(z_t, y_t) > 0$

Open economy facts

- What's new? The terms of trade, real exchange rate, and net exports.

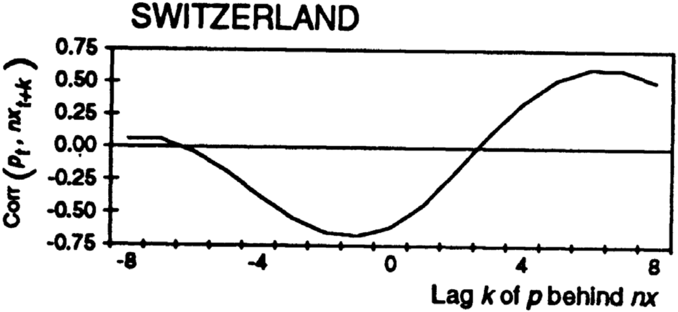
$$p = \frac{p_m}{p_x}$$
$$rer = \frac{p^*}{p}$$
$$nxy = \frac{ex - p_m \times m}{y}$$
$$rxy = \frac{ex - m}{y}$$

- New moments: cross-country correlations

Stylized facts: open economy

- High levels of comovement
  - $\rho(y_t, y_t^*) > \rho(c_t, c_t^*) > 0$
  - $\rho(\ell_t, \ell_t^*), \rho(x_t, x_t^*), \rho(z_t, z_t^*) > 0$
- Relative prices are volatile and persistent
  - $\sigma(rer_t) \approx 5 \times \sigma(y_t)$
  - $\rho(rer_t, rer_{t-1}) \approx 0.9$
- The J/S-curve: $\rho(p_t, nxy_{t+k})$ "lag k of p behind nxy"
  - $\rho(p_t, nxy_{t+k}) < 0$ when $k < 2$
  - $\rho(p_t, nxy_{t+k}) > 0$ when $k > 2$

J-curve


SWITZERLAND
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
Corr (p_t, nx_{t+k})
-8
-4
0
4
8
Lag k of p behind nx

Business cycle moments

- From BKK (1994), mostly 1950s–1990s

	Std. Deviation			Autocorrelation			Correlation		
	nx	y	p	nx	y	p	nx, y	nx, p	y, p
U.S.	0.45	1.83	2.92	0.80	0.82	0.80	–0.22	0.27	0.03
Median	1.06	1.53	2.92	0.71	0.74	0.80	–0.29	–0.46	0.03

	Std. Deviation rel. to y				XC correlation			
	c	x	p	nx	c, y	x, y	y_1, y_2	c_1, c_2
U.S.	0.45	3.15	1.59	0.25	0.76	0.90	0.70	0.46

Model

- Two countries $i = 1, 2$, representative agent
- Each country produces an intermediate good
- Intermediates combined to non-traded final good
- Preferences

$$V_{i0}(c) = \sum_{t=0} \sum_{s^t} \beta^t \pi(s^t) u(c_i(s^t), \ell_i(s^t))$$

- Complete markets (solve the planner problem)

Technology

- Country-specific goods: $i = 1$ gets a

$$y_i(s^t) = \exp(z_i(s^t))k_i(s^t)^\alpha \ell_i(s^t)^{1-\alpha}$$

- Nontraded final goods (symmetry, home bias)

$$D(a_1(s^t), b_1(s^t)) = (\omega^{\frac{1}{\gamma}} a_1(s^t)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} b_1(s^t)^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}}$$
$$D(b_2(s^t), a_2(s^t)) = (\omega^{\frac{1}{\gamma}} b_2(s^t)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} a_2(s^t)^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}}$$

Resource constraints

- Capital law of motion

$$k_i(s^{t+1}) = (1-\delta)k_1(s^t) + x_i(s^t)$$

- Intermediate goods

$$a_1(s^t) + a_2(s^t) = \exp(z_1(s^t))k_1(s^t)^{\alpha}\ell_1(s^t)^{1-\alpha}$$
$$b_1(s^t) + b_2(s^t) = \exp(z_2(s^t))k_2(s^t)^{\alpha}\ell_2(s^t)^{1-\alpha}$$

- Domestic absorbtion

$$c_1(s^t) + x_1(s^t) + g_1(s^t) = D(a_1(s^t), b_1(s^t))$$
$$c_2(s^t) + x_2(s^t) + g_2(s^t) = D(b_2(s^t), a_2(s^t))$$

Uncertainty

- Productivity and government spending follow AR(1)

$$z(s^t) = Az(s^{t-1}) + \epsilon_z(s^t)$$

$$g(s^t) = Bg(s^{t-1}) + \epsilon_g(s^t)$$

- A and B can have non-zero off diagonal terms
- ϵ are normal, mean zero, innovations can be correlated

$$\begin{bmatrix} \epsilon_z^1(s^t) \\ \epsilon_z^2(s^t) \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sigma_{\epsilon_z}^2 \begin{bmatrix} 1 & \rho_{\epsilon_z} \\ \rho_{\epsilon_z} & 1 \end{bmatrix} \right)$$

Calibration

- Technology: $\delta = 0.025$ and $\alpha = 0.36$
- Utility: $u(c, \ell) = (1-\sigma)^{-1}[c^{\mu}(1-\ell)^{(1-\mu)}]^{1-\sigma}$
  - $\beta = 0.99$ (interest rate), $\sigma = 2$ (given), choose μ to get ℓ_{ss}
- Armington aggregators

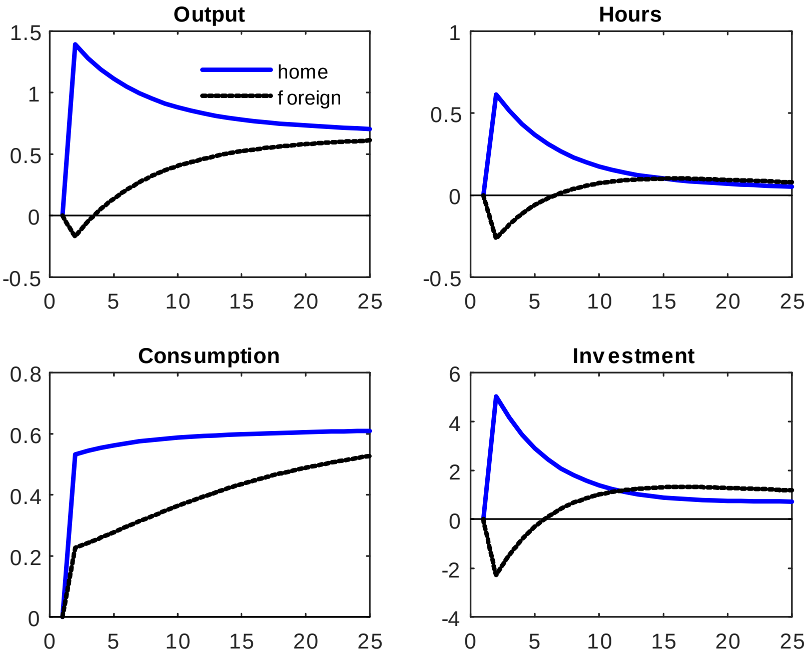
$$p = \frac{1-\omega}{\omega}\left(\frac{a_1}{b_1}\right)^{\frac{1}{\gamma}}$$

  - Symmetric steady state has $y_1 = y_2$, $b_1 = a_2$ and $p = 1$
  - γ given (estimates range from 0.25–20!)

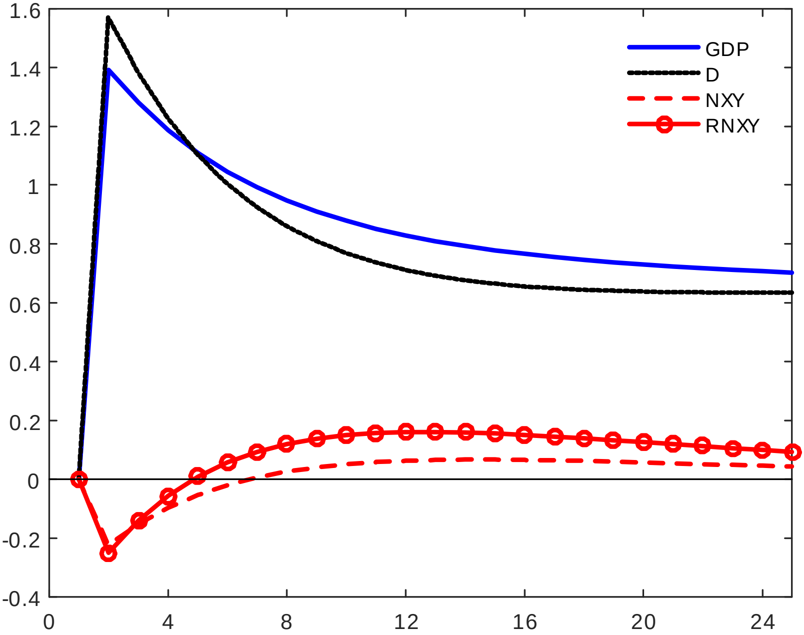
$$\left(\frac{\omega}{1-\omega}\right) = \frac{1-m/y}{m/y}$$

Shocks

- Estimated from Solow residuals
- $a_{11} = a_{22} = 0.906$ and $a_{12} = a_{21} = 0.088$
- $\sigma_{\epsilon z} = 0.085$ and $\rho_{\epsilon_z} = 0.258$
- Early literature focused on spillovers (technology diffusion?)

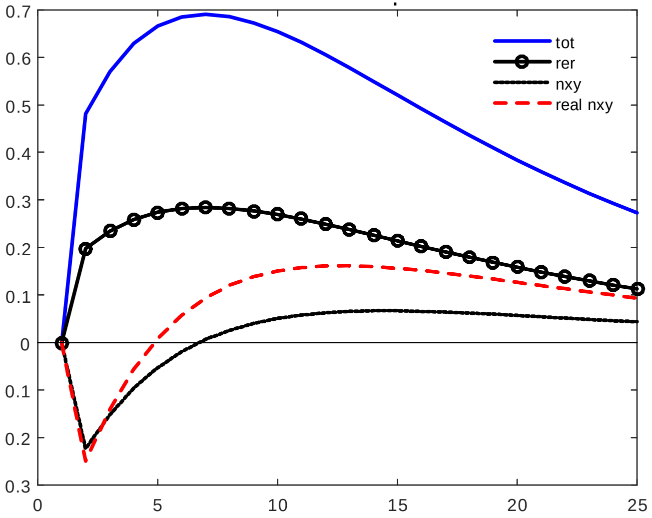


Output
home
foreign
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
Hours
1
0.5
0
-0.5
0
5
10
15
20
25
Consumption
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Investment
6
4
2
0
-2
-4
0
5
10
15
20
25



1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
4
8
12
16
20
24
GDP
D
NXY
RNXY

J/S-curve



J/S-curve

- Correlations depend on elasticity of substitution

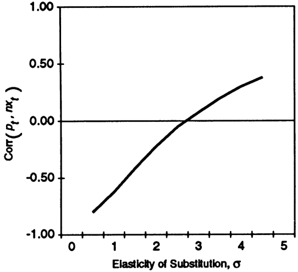


FIGURE 5. CORRELATION OF THE TRADE BALANCE AND THE TERMS OF TRADE FOR DIFFERENT VALUES OF THE ELASTICITY OF SUBSTITUTION

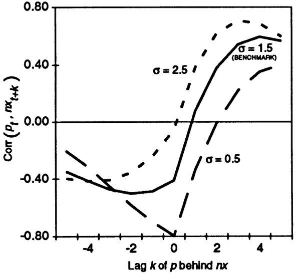


FIGURE 6. CROSS-CORRELATION FUNCTIONS WITH DIFFERENT ELASTICITIES

Demand shocks: government spending

- The twin deficits used to be an issue (1980s, 2000s)
- Calibrate government spending process
  - $g_{ss}/y_{ss} = 0.2$
  - $b_{11} = b_{22} = 0.95$, $b_{12} = b_{21} = 0.0$; $\sigma_g = 0.004$

Government spending

- Government steals consumption, borrow to smooth shock

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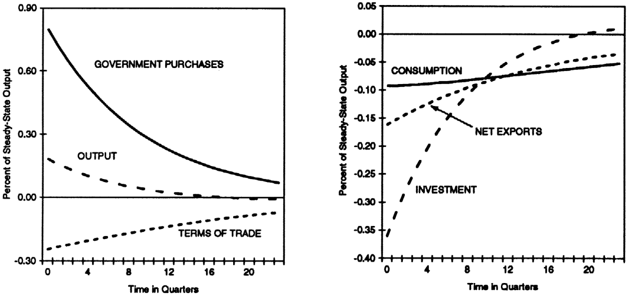
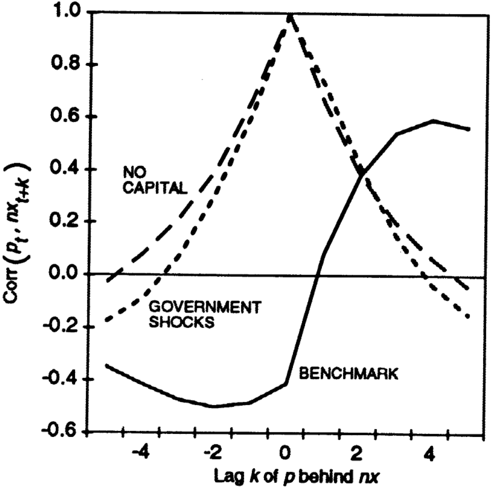


FIGURE 10. DYNAMIC RESPONSES TO A POSITIVE DOMESTIC GOVERNMENT SHOCK

Government spending

- Not a good theory of the current account



Business cycle moments

	Std. Deviation			Autocorrelation			Cross correlation		
	nx	y	p	nx	y	p	nx, y	nx, p	y, p
US	0.45	1.83	2.92	0.80	0.82	0.80	-0.22	0.27	0.03
Median	1.06	1.53	2.92	0.71	0.74	0.80	-0.29	-0.46	0.03
Models									
Bench.	0.30	1.38	0.48	0.61	0.63	0.83	-0.64	-0.41	0.49
2 shocks	0.33	1.33	0.57	0.62	0.65	0.78	-0.57	-0.05	
1 good	16.9	2.22	-	-0.10	0.76	-	0.10	-	-

- y and p are not volatile enough
- nx is too counter-cyclical
- p is too pro-cyclical

Business cycle moments

	Std. Deviation rel. to y				XC correlation			
	c	x	p	nx	c, y	x, y	y_1, y_2	c_1, c_2
US	0.45	3.15	1.59	0.25	0.76	0.90	0.70	0.46
Benchmark	0.47	3.48	0.35	0.22	0.88	0.93	0.02	0.77
Small elast	0.50	3.41	0.55	0.27	0.92	0.93	0.10	0.68
2 shocks	0.62	4.29	0.45	0.25	0.78	0.89	0.0	0.83
1 good	0.31	30.32	0	7.50	0.75	0.01	-0.58	0.46

- Consumption too correlated, output not correlated enough

International business cycle puzzles

1. Consumption/output anomaly: $\rho(c_t, c_t^*) > \rho(y_t, y_t^*)$ in model, opposite in data.

2. Price volatility: $\sigma(p_t)$ much too small in model versus data.

- True for real exchange rates, too

3. Backus-Smith puzzle: In the data $\rho\left(\Delta log(rer_t), \Delta log(c_t/c_t^*)\right) \approx 0$. Model predicts positive relationship.

4. Trade co-movement puzzle: In data, $\rho(y_t, y_t^*)$ is larger when trade between the two countries is larger. Much weaker relationship in the model.

From here…

- Backus-Kehoe-Kydland (AER 94): canonical international RBC
- Heathcote-Perri (JME 02): incomplete markets
- Raffo (JIE 08): quasi-linear preferences
- Alessandria-Choi (QJE 07), Ghironi-Melitz (QJE 05): hetero. firms
- Stockman-Tesar (AER 95): nontraded goods and preference shocks
- Backus-Crucini (JIE 00): oil
- Corsetti-Pesenti (QJE 01): new Keynesian IRBC
- Kose-Yi (JIE 06), Burstein-Kurz-Tesar (JME 08): trade and business cycle synchronization

Heathcote and Perri (JME 2002)

- "Financial autarky and international business cycles"
- Idea: Compute two-country, two-good model with different assumptions about financial markets
  1. Complete markets
  2. Non-contingent bond
  3. No asset trade
- Which matches the data the best?

Home country budget constraints

- Complete markets (market clearing for each b)

$$c(s^t) + x(s^t) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1}) = r(s^t)k(s^t) + \ell(s^t)n(s^t) + b(s^{t-1}, s_t)$$

- Non-contingent bond (market clearing for one b)

$$c(s^t) + x(s^t) + q(s^t)b(s^t) = r(s^t)k(s^t) + \ell(s^t)n(s^t) + b(s^{t-1})$$

- Financial autarky (trade balance)

$$c(s^t) + x(s^t) = r(s^t)k(s^t) + \ell(s^t)n(s^t)$$

Parameters taken from other studies		
Preferences	Discount factor	$\beta = 0.99$
	Consumption share	$\mu = 0.34$
	Risk aversion	$1 - \gamma = 2$
Technology	Capital share	$\theta = 0.36$
	Depreciation rate	$\delta = 0.025$
	Import share of *i*-firms (for calibrating ω_1)	$is = 0.15$
Estimated parameters	Productivity transition matrix[a]	$A = \begin{bmatrix} \underset{(0.007)}{0.970} & \underset{(0.008)}{0.025} \\ \underset{(0.008)}{0.025} & \underset{(0.007)}{0.970} \end{bmatrix}$
	Std. dev. of innovations to productivity	$\sigma_{\varepsilon 1} = 0.0073$ $\sigma_{\varepsilon 2} = 0.0044$
	Correlation of innovations to productivity	$corr(\varepsilon_1, \varepsilon_2) = 0.290$
	Elasticity of substitution between intermediate goods[b]	$\sigma = \underset{(0.12)}{0.90}$

(A) Volatilities[a]

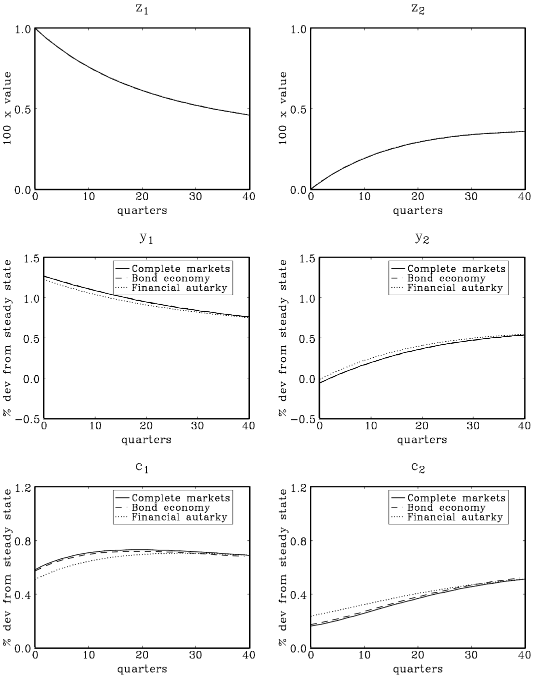
Economy	% std. dev. y	% std. dev. / % std. dev. of y c	x	n	% std. dev. ex	im	nx	ir
US data	1.67	0.81	2.84	0.66	3.94	5.42	0.45	4.07
Complete markets	1.21	0.53	2.74	0.31	0.99	0.99	0.20	0.70
Bond economy	1.21	0.52	2.73	0.32	0.96	0.96	0.19	0.76
Financial autarky	1.18	0.51	2.04	0.28	1.29	1.18	0.00	1.51

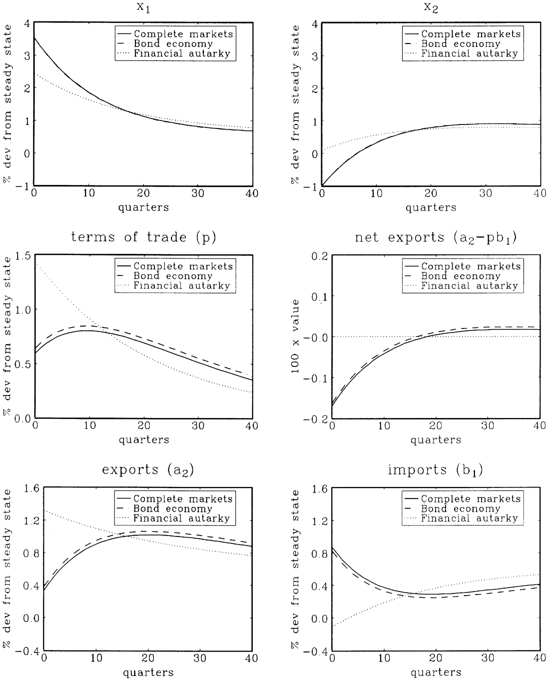
(B) Correlations with output[b]

Economy	correlation between c, y	x, y	n, y	ex, y	im, y	nx, y	p, y	rx, y
US data	0.86	0.95	0.87	0.32	0.81	− 0.49	− 0.24	0.13
Complete markets	0.96	0.96	0.97	0.55	0.89	− 0.64	0.65	0.65
Bond economy	0.95	0.96	0.97	0.59	0.86	− 0.65	0.65	0.65
Financial autarky	0.92	0.99	0.99	1.00	0.15	0.00	0.65	0.65

(C) Cross country correlations and international relative price volatility

Economy	correlation between y_1, y_2	c_1, c_2	x_1, x_2	n_1, n_2	% std. dev. p	rx
Data	0.58	0.36	0.30	0.42	2.99	3.73
Complete markets	0.18	0.65	0.29	0.14	0.78	0.55
Bond economy	0.17	0.68	0.29	0.17	0.84	0.59
Financial autarky	0.24	0.85	0.35	0.14	1.68	1.18


z1
z2
100 x value
quarters
y1
y2
% dev from steady state
Complete markets
Bond economy
Financial autarky
c1
c2



x1
x2
terms of trade (p)
net exports (a2−pb1)
exports (a2)
imports (b1)
Complete markets
Bond economy
Financial autarky
% dev from steady state
100 x value
quarters

Sensitivity to Armington elasticity

	Low persistence shocks $\rho = 0.95$ $\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 1.5$
(A) $corr(y_1, y_2) - corr(c_1, c_2)$			
Data	0.22		
Complete markets	0.13	– 0.13	– 0.30
Bond economy	– 0.37	– 0.14	– 0.18
Financial autarky	– 0.08	– 0.29	– 0.17
(B) $corr(x_1, x_2)$			
Data	0.30		
Complete markets	0.29	0.14	0.02
Bond economy	0.46	0.14	0.02
Financial autarky	0.66	0.61	0.46
(C) % std. dev. terms of trade (p)			
Data	2.99		
Complete markets	1.05	0.75	0.57
Bond economy	2.22	0.76	0.49
Financial autarky	5.74	1.41	0.80

Raffo (JIE 2008)

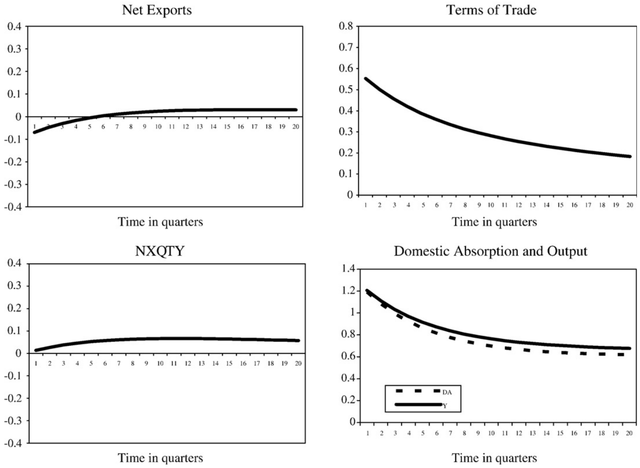
- "Net exports, consumption volatility, and international business cycle models"
- Idea: Why are (nominal) net exports countercyclical?
  - Real net exports are countercyclical
  - Terms of trade are acyclical (?)
- How does the benchmark model do in this regard?
- How can models better match data?

Country	Std Dev relative to output				Correlation with output		
	C	G	I	DA	NX	TOT	NXQTY
Australia	0.67	1.04	3.44	1.35	−0.36	−0.18	−0.40
Belgium	0.75	0.94	3.46	1.16	−0.18	−0.18	−0.05
Canada	0.77	0.62	2.62	1.12	−0.17	−0.22	−0.19
Finland	0.93	0.63	3.16	1.48	−0.27	−0.28	−0.46
France	0.82	0.78	2.85	1.07	−0.41	0.30	−0.18
Germany	0.85	0.90	2.08	1.05	−0.07	0.41	0.23
Italy	1.27	0.93	3.29	1.45	−0.27	−0.02	−0.29
Japan	0.67	0.84	2.53	1.05	−0.40	0.43	−0.14
Netherlands	0.99	0.79	2.87	1.11	−0.15	0.11	−0.14
Spain	1.03	1.21	3.48	1.69	−0.38	−0.06	−0.59
Sweden	1.08	0.87	3.68	1.03	−0.04	−0.28	−0.26
Switzerland	0.58	1.16	2.65	1.22	−0.19	−0.19	0.05
UK	1.16	0.79	3.31	1.22	−0.52	0.35	−0.32
US	0.74	0.68	2.74	1.05	−0.49	0.08	−0.44
EU-15	0.89	0.53	2.79	1.19	−0.54	0.16	−0.38
Average	0.88	0.85	3.00	1.21	−0.30	0.08	−0.24
Median	0.85	0.84	2.87	1.12	−0.27	0.11	−0.26

Note. DA = Domestic Absorption, C = Consumption, I = Investment, TOT = Terms of Trade, NX = Net Exports over GDP, NXQTY = Real Net Exports over Real GDP. All series were logged (except NX) and HP filtered using smoothing parameter of 1600.

- Nominal *nx* is countercyclical
- Real *nx* is countercyclical
- Terms of trade is ??
- Domestic absorbtion more volatile than output (by definition)

Standard BKK model



- Real *nx* is procyclical
- Countercyclical *nx* from terms of trade

Standard BKK model

BKK	Std Dev relative to output			Correlation with output		
	DA	C	I	NX	TOT	NXQTY
Data	1.12	0.81	2.76	−0.51	0.12	−0.41
Benchmark	0.98	0.58	2.76	−0.50	0.64	0.39
Large elasticity	0.95	0.56	2.76	0.26	0.64	0.50
Two shocks	0.96	0.71	2.76	−0.35	0.67	0.49
Bond economy	0.98	0.60	2.76	−0.53	0.63	0.32
Bond economy BC	0.99	0.62	2.76	−0.51	0.66	0.15
Bond economy CDL	1.02	0.65	2.76	−0.59	0.57	−0.19

Note. DA = Domestic Absorption, C = Consumption, I = Investment, TOT = Terms of Trade, NX = Net Exports over GDP, NXQTY = Real Net Exports over Real GDP. Statistics for the model refer to averages of 100 simulations of length 100 quarters after applying HP filter (smoothing parameter equal to 1600). In all simulations, capital adjustment costs are included to reproduce the volatility of investment relative to output.

- Real *nx* is procyclical
- Countercyclical *nx* from terms of trade
- Real *nx* driven by consumption that is too smooth

Increasing consumption volatility

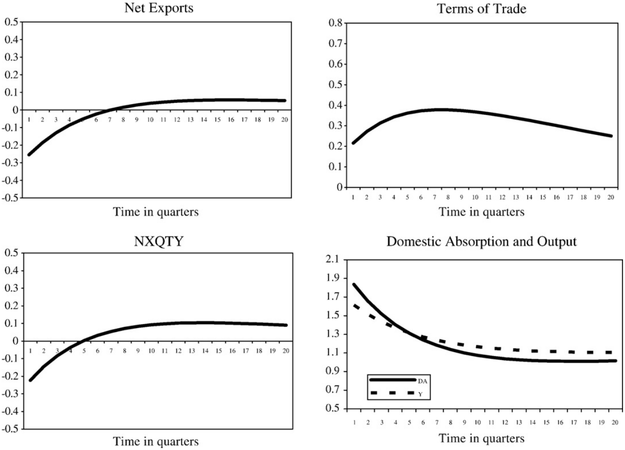
- Quasi-linear preferences (GHH (1988) preferences)

$$u(c, \ell) = \frac{(c - \psi \ell^{\nu})^{1-\gamma}}{1-\gamma}$$

- First order condition implies

$$\psi \nu \ell^{\nu - 1} = w$$

BKK + GHH



- Real *nx* is countercyclical
- Terms of trade still procyclical, but dampened

BKK + GHH

Symmetric BKK with GHH preferences

	Std Dev relative to output			Correlation with output		
	DA	C	I	NX	TOT	NXQTY
Data	1.12	0.81	2.76	−0.51	0.12	−0.41
BKK	0.98	0.58	2.76	−0.50	0.64	0.39
GHH	1.09	0.79	2.76	−0.51	0.43	−0.44

- Model with GHH closer to data