

## ECON 871 Problem set #1

This exercise is meant to get you comfortable working with data and solving heterogeneous-firm models. You are welcome to discuss the work with others, but whatever you turn in should be your own work. Please keep the results to less than 10 pages. It is important to present results in a clean and concise manner.

1. **Data.** Use the census data set from [Colombia](#) to calculate some cross-sectional and dynamic moments related to producers and trade. The unit of observation is a plant. Drop the years from the dataset that do not have exports reported.
  - (a) **Plant size distribution, birth, and death.** (i) Calculate the mean and variance of the employment size distribution across all plants. (ii) Recalculate these moments for continuing plants, newborns (i.e., plants that were not in the dataset in the previous period), and exiting plants (i.e., plants that are not in the dataset in  $t+1$ ). (iii) Calculate the birth and death rates (as a share of existing firms).
  - (b) **Exporter size premia.** Estimate the exporter premium, (i.e., the gap in size between exporters and non-exporters) in terms of sales, capital, employment, and materials.
  - (c) **Plant-level dynamics.** Estimate an AR(1) processes for log: employment, total sales, domestic sales, and exports.
  - (d) **Exporter moments.** (i) Calculate the share of firms that export. (ii) Among non-exporters, calculate the export entry rate. (iii) Among exporters, calculate the export stopper rate and (iii) mean export intensity (exports/total shipments).
2. **The sunk-cost model.** Consider the canonical partial-equilibrium sunk-cost model discussed in class. Assume there are two symmetric countries with a unit mass of firms in each country. There is no firm birth or death. At the beginning of each period the firm knows its productivity ( $a$ ) and faces a fixed cost of exporting,  $f$ . The firm's productivity follows an AR(1) process in logs

$$\log a_{t+1} = (1 - \rho) \log \bar{a} + \rho \log a_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1).$$

The fixed cost depends on the previous period's export participation decision. It equals  $f_0$  if the firm did not pay a fixed export cost in the previous period and equals  $f_1$  if the firm did pay a fixed export cost in the previous period. Paying a fixed export cost today allows the firm to export the next period with a variable export cost ( $\xi \geq 1$ ), otherwise it cannot export.

Firms discount future profits according to a constant discount factor  $0 \leq \beta < 1$ . Firms face the same demand curves at home and abroad,  $d(p) = p^{-\theta}$  but also incur a shipping cost,  $\xi$ , on exports and foreign consumers pay a tariff,  $\tau \geq 1$ . The static profit of the firm is

$$\begin{aligned} \pi(a, X) &= \max_{p, p^*, l} p^{1-\theta} + Xp(p^*\tau)^{-\theta} - wl \\ \text{s.t. } & p^{-\theta} + X(\tau p^*)^{-\theta} \xi = y + x = al \end{aligned}$$

where  $X$  is equal to one if the firm paid a fixed cost in last period and zero otherwise.

The firm's bellman equation is

$$V(a, X) = \pi(a, X) + \beta \max \{-f_0(1 - X) - f_1(X) + \beta EV(a', 1), \beta EV(a', 0)\}$$

- (a) Set the parameters to:  $\theta = 4$ ,  $\bar{a} = 1$ ,  $\beta = 0.9$ ,  $\tau = 1.1$ ,  $w = 1.14$ ,  $\rho = 0.92$ ,  $\sigma = 0.25$ ,  $\xi = 1.2$ ,  $f_0 = 0.40$ , and  $f_1 = 0.28$ .

Solve the model for the stationary equilibrium. Using whatever technique you prefer (simulating paths, solving for stationary distributions and computing, something else?) compute:

- i. Export participation rate
- ii. Export stopper rate
- iii. Exporter domestic sales premium
- iv. Mean export intensity among exporters
- v. Autocorrelation of domestic sales.

- (b) Plot the export exit and entry policy as function of  $a$  and  $X$ .

- (c) Derive a relationship between the starter rate, the stopper rate, and the export participation rate in the model's stationary equilibrium. Does this relationship hold in your data? What does this imply?

**A note on programming.** You will use the code you write for this question to estimate some of the model parameters in the next problem set. When you plan out your code for this problem, keep in mind that in the next problem set you will be solving the model and computing moments (basically, question [2a](#)) for arbitrary parameter vectors.