# Trade and uncertainty

[*My notes are in beta. If you see something that doesn't look right, I would greatly appreciate a heads-up.*]

Models with sunk costs and uncertainty—regardless of the application—have the feature that uncertainty creates an incentive to delay paying the sunk cost. This idea has had a resurgence lately in the uncertainty-shock theory of investment during recessions. The idea is that uncertainty increases during recessions so firms delay making investments. This generates a drop in investment, even if nothing bad has happened. The extra uncertainty is enough. Nick Bloom has written extensively on this topic.

This idea has recently been taken up in international trade. The key features are already in place

1. There is an option to invest (enter the export market) in the future rather than today
2. The value of the investment (the sunk entry cost) cannot be recovered in the future

The trade applications have largely focused on China's accession to the WTO, but the current trade war between the United States and China and the potential Brexit have the same features.

## A simple example

Suppose a firm is faced with an opportunity to invest in a piece of capital that will produce a good whose return today is 200, but in the future, it may be either 350 or 50 per year. To keep things simple, the value of the return will be known for certain next year, and once the new value is known, it will stay that way forever. Also assume that the investment, once it is made, has no outside value: It cannot be resold. The price of the machine is 1,100. Should the firm fund the project?

To answer this question, we need to know more about the future returns. Let $q_L = 0.5$ be the probability of that the return is 50 and let $q_H = 0.5$ be the probability that the return is 350. Notice that $q_L + q_H = 1$, and since they are each equal to 0.5, the probability distribution says that both events are equally likely to happen. The potential paths of returns are illustrated in figure 1.

Figure 1: Uncertainty

Add a figure of the uncertainty tree.

Now that we know the potential returns on the investment, we can compute the net present value (NPV) of investing in the machine. Should the firm invest today, if the risk free interest rate is $r = 0.10$? The NPV is the expected return ($q_L p_L + q_H p_H$) per period, discounted by the risk free interest rate, after subtracting off the cost of the investment

$$NPV_0 = -c + p_0 + \sum_{t=0}^{\infty} \frac{p_t}{(1+r)^t} = -c + p_0 + \sum_{t=1}^{\infty} \frac{1}{(1+r)^t}(q_L p_L + q_H p_H) \tag{1}$$

$$NPV_0 = -1100 + 200 + \sum_{t=1}^{\infty} \frac{1}{(1.10)^t}(0.5 \times 50 + 0.5 \times 350) = 1100$$

The NPV is positive, so this seems like a good investment. We are missing something, however. By investing today, the firm faces the downside risk of owning a machine that produces low returns. What should the firm do if it could choose between investing today or, waiting until next year, and investing once the value of $p$ is known?

## The value of waiting

Let's extend our example to allow the firm to either invest today or tomorrow. If the firm invests today, it earns today's return of 200 plus the future returns. If the firm waits until next period, when it knows whether the return will be low or high, the firm avoids investing in a machine with a low return. Let's compute the value of the three possible actions: investing today (done!), investing tomorrow if the price is 350, and investing tomorrow if the price is 50.

If the firm waits until next period and invests only if the project is good, the NPV is

$$NPV_1^H = q_H \times \left[\frac{-c}{1+r} + \sum_{t=1}^{\infty} \frac{p_H}{(1+r)^t})\right] + q_L \times 0 \tag{2}$$

$$NPV_1^H = 0.5 \times \left[\frac{-1100}{1.1} + \sum_{t=1}^{\infty} \frac{350}{(1.1)^t}\right] + 0.5 \times 0 = 1250 \tag{3}$$

Notice that compared to (1) we discount the cost of the investment, because we do not pay it until next period, and that we do not earn the current period return, $p_0$. The NPV from waiting until next period and investing in the high return state is 1250, compared to 1100: The value of waiting to invest is 150. The economic idea embodied in that 150 is that by not committing today, we have the option of passing on a bad investment once we know more about the future payoffs.

We can check that investing in the low return state is not worth doing,

$$NPV_1^L = q_H \times 0 + q_L \times \left[\frac{-c}{1+r} + \sum_{t=1}^{\infty} \frac{p_L}{(1+r)^t}\right] \tag{4}$$

$$NPV_1^L = 0.5 \times 0 + 0.5 \times \left[\frac{-1100}{1.1} + \sum_{t=1}^{\infty} \frac{50}{(1.1)^t}\right] = -250 \tag{5}$$

The idea that there is value in having flexibility about when an investment (or many other decisions) is made is known as a *real option*. You would be willing to pay 150 for the option of making the investment a year from now. Dixit and Pindyck (1994) is the canonical exposition of the real option approach.

## Options and Changes in Uncertainty

When faced with uncertain payoffs or costs, delaying a decision can be optimal. How does the option value change as the amount of uncertainty increases?

Let's return to our example above. Our original set of outcomes $(p_L = 50, p_H = 350)$ and probabilities $(q_L = 0.5, q_H = 0.5)$ implies that the standard deviation of the return is 106.1. Now, suppose that the project has become more risky: The probabilities of a good and bad outcome stay the same, but the outcomes change, $(p_L = 25, p_H = 375)$. This is a *mean preserving spread*: The expected value of the project return has stayed the same (200), but the standard deviation has increased to 123.7. Since the expected value of the project has not changed, the NPV of investing at $t = 0$ says the same. What is the value to waiting until next year and investing if the project turns out to be of high quality?

$$NPV_1^H = 0.5 \times \left[ \frac{-1100}{1.1} + \sum_{t=1}^{\infty} \frac{375}{(1.1)^t} \right] + 0.5 \times 0 = 1375. \tag{6}$$

The value to waiting and investing has increased from 150 to 275. Why does the value of waiting increase with volatility? As volatility increases, both the upside gain and the downside risk of the project increase. If you invest today, you face both the upside gain and the downside risk. By waiting, the downside risk remains at zero, but the larger upside gain remains.

Notice that we have said nothing about risk aversion. While risk aversion may also make people reluctant to invest in risky outcomes the real option concept does not require the decision maker to be risk averse. A risk neutral investor—who cares only about the expected return of a project—would still prefer to wait-and-see rather than invest early.

## Handley (2014)

One of the big points here is that lowering the current tariff without lowering the bindings creates more uncertainty and attenuates the effect of lower tariffs.

## The WTO

- Bargain over a bound tariff. A bound tariff says that the country cannot increase the tariff above this rate. They can decrease it if they like.
- Many countries have goods with tariffs below their bound rates. The difference between the current tariff and the bound tariff is the potential extra tariffs an exporter may face.
- Show figures 1 and 2.

## Uncertainty

- In each period, tariff change with probability $\gamma$
- When the tariff change arrives, a new tariff, $\tau'$ is drawn from (CDF) $H(\tau)$
- The long-run autocorrelation is $1-\gamma$

## Entry and exit

- Export entry is once-and-for-all. An exporter only exits when there is exogenous death to the firm, which happens with probability $\delta$. The value of an exporter is $V^1$.
- A nonexporter enters the export market when $V^1 - K_e \geq V^0$

**Value of exporting**

Value of exporting. I never voluntarily leave the export market. $\beta = (1-\delta)/(1-\rho)$ includes both discounting and the death probability.

$$V^1(c,\tau_t) = \pi(c,\tau_t) + \beta\left[(1-\gamma)V^1(c,\tau_t) + \gamma \mathop{\mathbb{E}}_{\tau'} V^1(c,\tau')\right] \tag{7}$$

Suppose the tariff change event has occurred. What is the expected value of being an exporter before you learn the value of $\tau$?

$$\mathop{\mathbb{E}}_{\tau'} V^1(c,\tau') = \mathop{\mathbb{E}}_{\tau'} \pi(c,\tau') + \beta\left[(1-\gamma)\mathop{\mathbb{E}}_{\tau'} V^1(c,\tau') + \gamma \mathop{\mathbb{E}}_{\tau'}\mathop{\mathbb{E}}_{\tau''} V^1(c,\tau'')\right] \tag{8}$$

As long as $H(\tau)$ is time-invariant, the last term is just the expectation. Solve this for $\mathbb{E}_{\tau'} V^1(c,\tau')$ — the ex ante expected profit from exporting when a tariff change will occur — to yield

$$\mathop{\mathbb{E}}_{\tau'} V^1(c,\tau') = \frac{\mathbb{E}_{\tau'}\,\pi(c,\tau')}{1-\beta}. \tag{9}$$

This is just the present value of earning the expected period export profit. Substitute (9) back into (8) and solve for $V^1$:

$$V^1(c,\tau_t) = \pi(c,\tau_t) + \beta\left[(1-\gamma)V^1(c,\tau_t) + \frac{\gamma}{1-\beta}\mathop{\mathbb{E}}_{\tau}\pi(c,\tau')\right]$$

$$V^1(c,\tau_t) = \frac{(1-\beta)\pi(c,\tau_t) + \beta\gamma\,\mathbb{E}_\tau\,\pi(c,\tau')}{(1-\beta)(1-\beta(1-\gamma))} \tag{10}$$

**Value of not exporting**

For each value of $c$, there exists a value for $\tau$ such that the firm will enter the export market. Call this value $\tau_1$. [I do not see how this can be true for any $c$. There should be firms that are bad enough that they will not enter the export market if $\tau = 1$.] The value of not exporting is

$$\begin{aligned} V^0(c, \tau_t) &= 0 + \beta(1-\gamma)V^0(c, \tau_t) \\ &\quad + \beta\gamma\left(1 - H(\tau_1)\right) \mathop{\mathbb{E}}_{\tau'|\tau'>\tau_1} V^0(c, \tau') \\ &\quad + \beta\gamma H(\tau_1)\left[\mathop{\mathbb{E}}_{\tau'|\tau'\leq\tau_1} V^1(c, \tau') - K_e\right] \end{aligned} \tag{11}$$

(This is different than in the paper.) The first term is zero: a nonexporter earns no export profits in the current period. The second term is the value if no tariff change happens. If the firm is not exporting currently at $\tau_t$ then it will not export in the future at $\tau_t$. The third term is the value of the tariff changing, but the new tariff not being low enough to induce the firm to enter. The fourth term is the value of the tariff changing and the new tariff being low enough that the firm enters.

The last term depends on the expected value of exporting conditional on entering.

$$\mathop{\mathbb{E}}_{\tau'|\tau'\leq\tau_1} V^1(c, \tau') = \mathop{\mathbb{E}}_{\tau'|\tau'\leq\tau_1} \pi(c, \tau') + \beta\left[(1-\gamma)\mathop{\mathbb{E}}_{\tau'|\tau'\leq\tau_1} V^1(c, \tau') + \gamma \mathop{\mathbb{E}}_{\tau'|\tau'\leq\tau_1} \mathop{\mathbb{E}}_{\tau''} V^1(c, \tau'')\right] \tag{12}$$

**Solution**

According to Handley, equations (9), (10), (11) and (12) are a system four equation in four unknowns $V^1(c, \tau_t)$, $\mathbb{E}_{\tau'} V^1(c, \tau')$, $V^0(c, \tau_t)$, and $\mathbb{E}_{\tau'|\tau'\leq\tau_1} V^1(c, \tau_1)$. I think we need the expected value of not exporting when $\tau$ changes but $\tau' > \tau_1$. Maybe it drops out of the math. I need to think about this some more.

With enough work, you can solve these equations analytically. If we take $V^1(c, \tau_1) - V^0(c, \tau_1) = K_e$, we have the break-even condition. We can solve this for the $c$ that makes this true for $\tau_1$. We wrote down the problem as a fixed $c$ implies some $\tau_1$. Now we are asking, for a given $\tau_1$, what firm type is break-even?

This is a good equation.

$$K_e = \frac{\pi(c^U, \tau_1)}{1-\beta(1-\gamma)} + \frac{\beta\gamma\,\mathbb{E}\,\pi(c^U, \tau')}{(1-\beta)[1-\beta(1-\gamma)]} + \frac{\beta\gamma H(\tau_1)\left[\pi(c^U, \tau_1) - \mathbb{E}_{\tau'<\tau_1} \pi(c^U, \tau')\right]}{(1-\beta)[1-\beta(1-\gamma)]} \tag{13}$$

$$K_e = V^1(c^U, \tau_1) - \frac{\beta\gamma H(\tau_1)\left[\mathbb{E}_{\tau'<\tau_1} \pi(c^U, \tau') - \pi(c^U, \tau_1)\right]}{(1-\beta)[1-\beta(1-\gamma)]} \tag{14}$$

The first term is the value of being an exporter when $\tau = \tau_1$. The second term is the opportunity cost of entering today at $\tau_1$, rather than waiting and entering when $\tau < \tau_1$.

Insert the expression for $\pi(c, \tau) = A\tau^{-\sigma}c^{1-\sigma}$ into the equation and solve for $c^U$

$$c^U = \left[\frac{A\tau^{-\sigma}}{(1-\beta)K_e}\right]^{\frac{1}{\sigma-1}} \left[\frac{1-\beta+\beta\gamma\Delta(\tau_1)}{1-\beta+\beta\gamma}\right] \tag{15}$$
$$c^U = c^D \left[\frac{1-\beta+\beta\gamma\Delta(\tau_1)}{1-\beta+\beta\gamma}\right] \tag{16}$$

The term in brackets, which Handley calls $\Theta(\tau)$ is less than one. This means that the firm in an uncertain environment must be a **better firm** — have lower costs/higher productivity — to enter compared to that in a deterministic environment.

So $c^U$ is the firm type that is indifferent to entering at the tariff $\tau_1$. As we vary $\tau_1$, we change the cutoff type.

The important part here is

$$\Delta(\tau_t) - 1 = (1 - H(\tau_t))\frac{\mathbb{E}_{\tau\geq\tau_t}\,\tau^{-\sigma} - \tau_t^{-\sigma}}{\tau_t^{-\sigma}} \tag{17}$$

The first term is the probability that you have a higher tariff in the next tariff change. The second term is the expected loss (look at the profit function definition) from the higher tariff. The loss is always negative, unless $\tau_t = \tau_{max}$.

## How trade agreements matter

1. Trade agreements may decrease the chance that tariffs change, a decrease in $\gamma$
2. Trade agreements may bind tariffs so that they can only increase by so much

Handley looks at the second. Let $B < \tau_{\text{max}}$ be the bound tariff level.

$$\Delta(\tau_t, B) - 1 = (1 - H(B))\frac{B^{-\sigma} - \tau_t^{-\sigma}}{\tau_t^{-\sigma}} + (H(B) - H(\tau_t))\frac{\mathbb{E}_{\tau_t\leq\tau\leq B}\,\tau^{-\sigma} - \tau_t^{-\sigma}}{\tau_t^{-\sigma}} \tag{18}$$
$$= (1 - H(B))\frac{B^{-\sigma} - \mathbb{E}_{\tau>B}\,\tau_t^{-\sigma}}{\tau_t^{-\sigma}} + [\Delta(\tau_t) - 1] \tag{19}$$

The first term is the profit loss avoided by the binding. When $B = \tau_{\text{max}}$, the bindings do nothing. When $B = \tau_t$ there is no downside risk and $C^U = C^D$.

**Proposition 1**

$$\frac{d \log c_t^U}{d \log B} < 0 \tag{20}$$

$$\frac{d \log c_t^U}{d \log \gamma} \leq 0 \tag{21}$$

$$\frac{d^2 \log c_t^U}{d \log \gamma \, d \log B} \leq 0 \tag{22}$$

Lowering $B$ makes it easier to enter. Lowering $\gamma$ makes it easier to enter. Lower bindings make $\gamma > 0$ less harmful to entry.

**Proposition 2**

$$\frac{d \log c_t^U}{d \log \tau_t} = \frac{d \log c_t^D}{d \log \tau_t} + \frac{d \log \Theta_t}{d \log \tau_t} \tag{23}$$

The second term on the right is less than or equal to 0, so the entry elasticity when there is uncertainty is lower than in the deterministic case.

---

Dixit, Avinash K. and Robert K. Pindyck (1994). *Investment under Uncertainty*. Princeton University Press.